Comments On Coupling Correction in Tevatron

Is there a problem ?

♦ There is still an appreciable beat of betatron amplitudes at 150 (though significantly smaller than before September 2003 shutdown) which indicates that coupling is still strong

♦ CDF reports large (increased after the last shutdown) correlation $<x \cdot y>$ within the luminous region

Questions to answer :

♦ Why there is beat of betatron amplitudes no matter what the minimum tunesplit is?
(answer – it is determined by both difference and sum resonance driving terms)

♦ What part of coupling produces beam ellipse tilt at IPs?

♦ How coupling manifests itself in TBT spectra and differential orbits?

♦ How to define global and local corrections in terms of the difference and sum resonance driving terms?

♦ Is there enough skew-quad circuits for global and local correction of the difference and sum resonances?

♦ How to avoid involuntarily enhancing the sum resonance while reshimming the dipoles to eliminate the skew-quad component?

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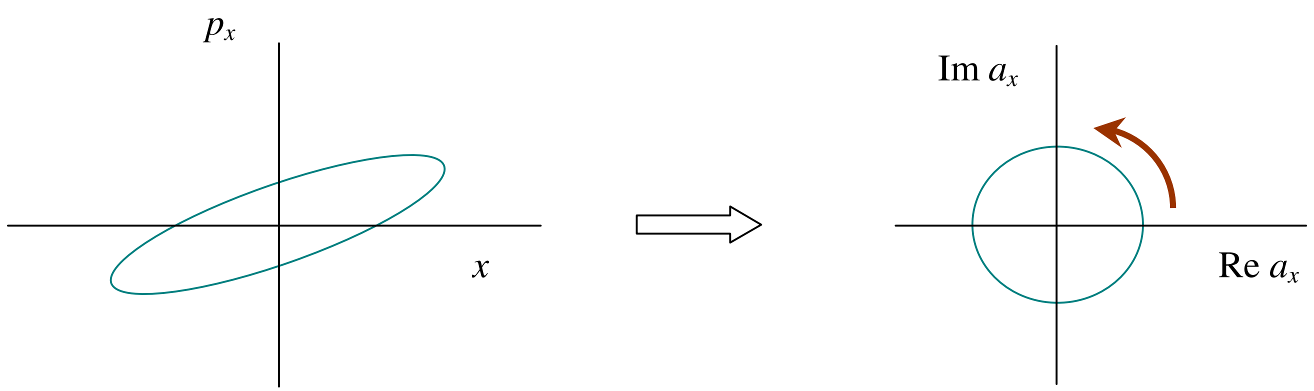
Theoretical excursion: normal forms



Uncoupled optics: using the Twiss parameters α_x, β_x and periodic phase advance ϕ_x

$$a_x = \frac{1}{\sqrt{2\beta_x}}[(1-i\alpha_x)x - i\beta_x p_x]\exp[-i\,\underbrace{(\varphi_x - Q_x\theta)}_{\phi_x}]$$

Conversely $\quad x = \sqrt{2\beta_x}\,\mathrm{Re}(a_x e^{i\phi_x})$

Unperturbed motion:

$$a_x = \sqrt{I_x}\exp(i\,Q_x\theta + i\psi_{x0})$$

I_x being the action invariant, $\theta = s/R$ advances by 2π/turn

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How to measure a_x ?

Two BPMs are necessary - we cannot measure the momentum directly. At *k*-th turn:

$$a_x = \frac{i}{\sin(\varphi_{x2} - \varphi_{x1})}\left(\frac{x_{1k}}{\sqrt{2\beta_{x1}}}e^{-i\varphi_{x2}} - \frac{x_{2k}}{\sqrt{2\beta_{x2}}}e^{-i\varphi_{x1}}\right)\times e^{iQ_x(\theta - 2\pi k)}$$

Description of coupling

Tilt of the normal modes

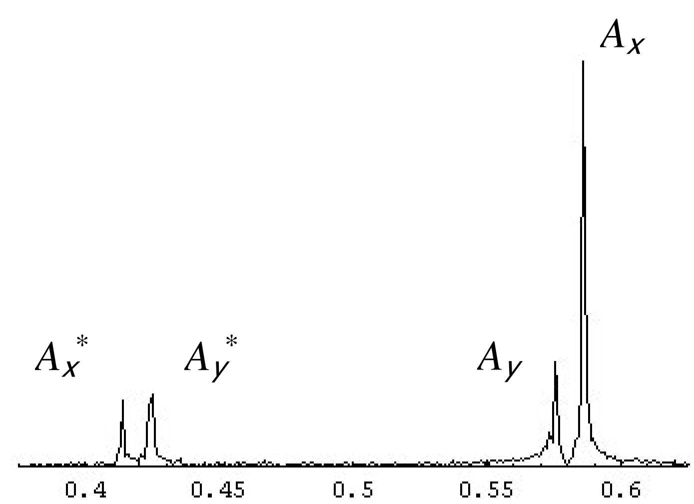
Linear approximation in couplers strength

$$a_x \approx A_x + w_-^* A_y + w_+^* A_y^*$$

$$a_y \approx A_y - w_- A_x + w_+^* A_x^*$$

$A_x \sim e^{iQ_1\theta}$, $A_y \sim e^{iQ_2\theta}$ - "true" normal forms, both present in all BPMs

These equations show how w_- and w_+ can be extracted from spectra of TBT oscillations



Fourier spectrum of a_x calculated from 2 BPMs TBT data. A_x^* appeared due to optics perturbations, A_y and A_y^* are manifestation of coupling (w_- and w_+ correspondingly).

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One more piece of theory

Both w_- and w_+ originate from the same couplers (though produce different effects)

$$w_{\pm}(\theta) = -\int_0^{2\pi} \frac{\exp\{-i(Q_x \pm Q_y)[\theta - \theta' - \pi\,\mathrm{sgn}(\theta - \theta')]\}}{4\sin\pi(Q_x \pm Q_y)} C_{\pm}(\theta')d\theta'$$

$$C_{\pm}(\theta) = \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\left\{\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right) + B_\theta\left[\left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\right) - i\left(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\right)\right]\right\}\exp[i(\phi_x \pm \phi_y)]$$

Dominant contribution from harmonics:

$C_-(\theta) \approx C_-^{(0)}$, closest tune approach = $|C_-^{(0)}|$

$C_+(\theta) \approx C_+^{(-n)}$, n = Integer($Q_x + Q_y$) = 41

Correspondingly

$$w_-(\theta) \approx -\frac{C_-}{2(Q_{x0} - Q_{y0})} = const$$

$$w_+(\theta) \approx -\frac{C_+^{(-n)}e^{-in\theta}}{2(Q_{x0} + Q_{y0} - n)}$$

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Observable effects of coupling

Tuneshifts:

$$Q_1 - Q_2 \approx \sqrt{(Q_{x0} - Q_{y0})^2 + |C_-|^2} \qquad \text{almost independent of } C_+$$

$$Q_1 + Q_2 \approx n + \sqrt{(Q_{x0} + Q_{y0} - n)^2 - |C_+|^2} \qquad \text{almost independent of } C_-$$

Betatron amplitude beat:

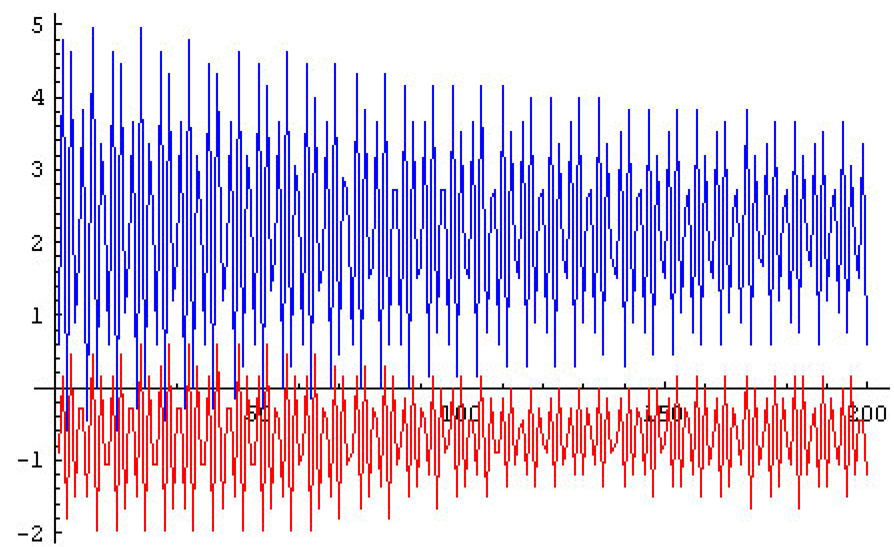
Comes from:
- Normal modes tilt at the kicker and BPM locations
- Actual tilt of the kicker and BPMs – presumed to be small

Average over fast beats

$$\left\langle |a_y|^2 / |a_x|^2 \right\rangle \approx 2|w_+|^2 +$$

$$+ 2|w_-|^2 \, [1 - \cos(2\pi \Delta Q \cdot k)]$$

- permits to make an estimate of both $|w_-|$ and $|w_+|$ but not their phases.



TBT data from HE22 (blue) and VE21 (red) BPMs after a horizontal kick on the central orbit. Coupling corrected to $|C_-|$<0.004

Comments On Coupling Correction in Tevatron

Effect on the closed orbit

Let at $\theta = \theta_k$ be a corrector (or separator) providing horizontal kick χ

$$\Delta a_x = -i\chi\sqrt{\frac{\beta_x(\theta_k)}{2}}e^{-i\phi_x(\theta_k)}, \quad \Delta a_y = 0 \quad \Rightarrow$$

$$\Delta A_x \approx \Delta a_x, \quad \Delta A_y \approx w_-(\theta_k)\Delta a_x - w_+^*(\theta_k)\Delta a_x^*$$

Off-plane closed orbit:

$$a_y(\theta) \approx \frac{w_-^{(0)}(Q_1 - Q_2)}{2\sin\pi Q_1}(\theta - \theta_k - \pi)e^{iQ_1(\theta-\theta_k-\pi)}\Delta a_x +$$

$$+\frac{w_+^{(n)}(Q_1 + Q_2 - n)}{2\sin\pi Q_1}(\theta - \theta_k - \pi)e^{-iQ_1(\theta-\theta_k-\pi)+in\theta}\Delta a_x^*$$

- significantly smaller than the projection of mode 1 free oscillations on the vertical plane since the tunes are close to half-integer (far from the integer resonance).

This means that TBT is (in principle) more sensitive method of coupling detection than the differential orbit.

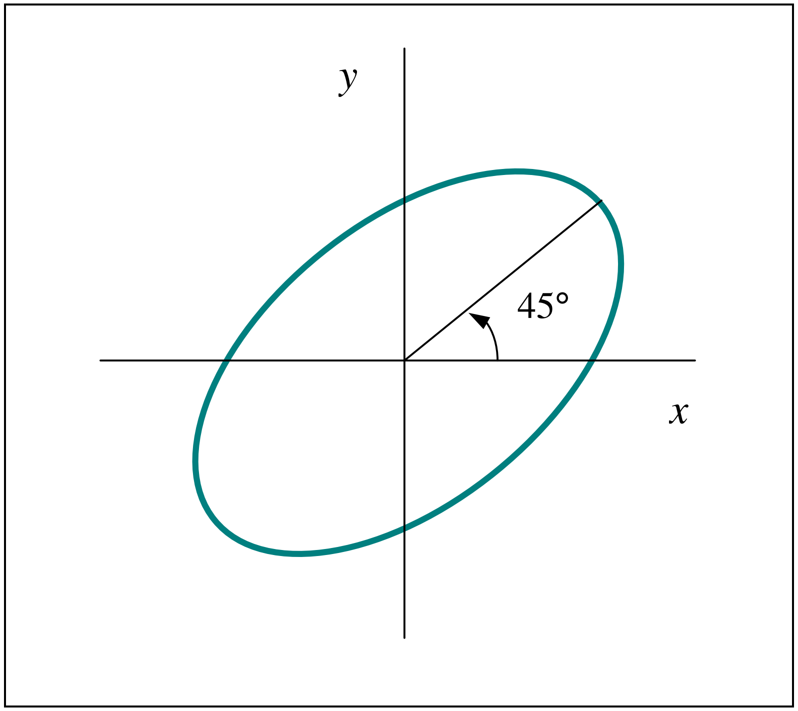
Comments On Coupling Correction in Tevatron

Eccentricity of the beam ellipse at IP

- in the case of equal emittances does not depend on w_-
- in the case of symmetrical optics and equal β-functions

$$\sigma_{\min} = \frac{\sigma_0}{\sqrt{1 + 2\,|\,\mathrm{Re}\, w_+\,|}}, \qquad \sigma_{\max} = \frac{\sigma_0}{\sqrt{1 - 2\,|\,\mathrm{Re}\, w_+\,|}},$$

$$< x \cdot y > \approx 2\,\mathrm{Re}\, w_+ \cdot \sigma_0^2$$



Vertical dispersion

- an important issue at injection, but it is not addressed in the present note

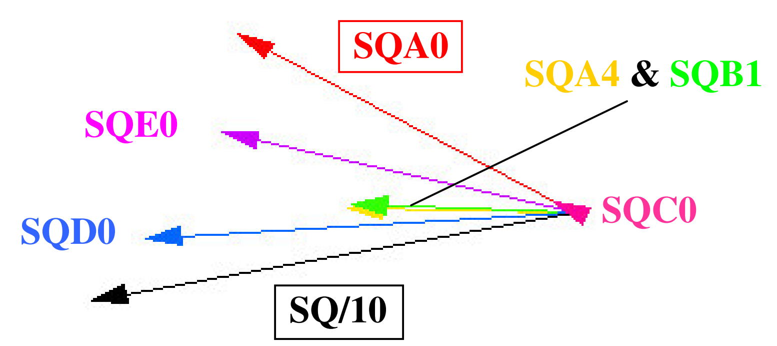
Comments On Coupling Correction in Tevatron

Coupling correction:

Local: $w_{\pm}(\theta) \rightarrow 0$ at a particular location

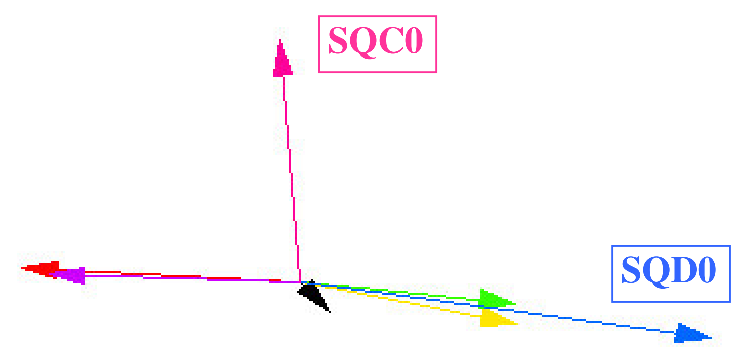
Global: $w_{-}^{(0)} \rightarrow 0$, $w_{+}^{(-n)} \rightarrow 0$

w_{-} correction: local=global



Contribution to w_{-} from different skew-quad circuits at 150 (SQ with 10 times smaller K1L/quad than the others). SQ and SQA0 are known to be the most efficient circuits (though not completely orthogonal)

w_{+} global correction at injection:



Contribution to $w_{+}^{(-n)}$ from different skew-quad circuits at 150. SQC0 is a proposed circuit consisting of two unused skew-quads around C0 connected with opposite polarity. It forms an almost orthogonal pair with SQD0.

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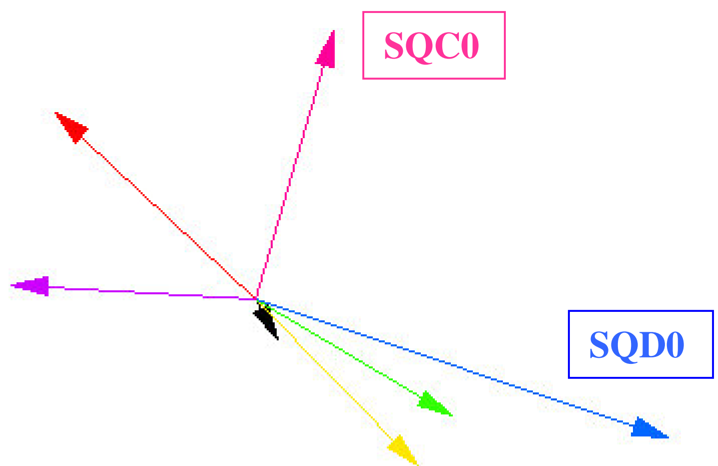
w_+ local correction at injection:



Contribution to w_+ at MTEV60 (mid-point of the F0 Lambertsons array) from different skew-quad circuits at 150 GeV (the same colors as in the previous two plots)

w_+ local correction at low-beta:

circuit	B0	D0
SQ/10	-0.00006+0.00006i	-0.00007+0.00004i
SQA0	0.00038 -0.00043i	0.00057+0.00047i
SQA4	-0.00177+0.00088i	-0.00177+0.00089i
SQB1	-0.00183+0.00075i	-0.00177+0.00089i
SQD0	-0.00332	-0.00337 -0.00152i
SQE0	0.00051 -0.00001i	0.00031+0.00040i
SQC0	0.00009 -0.00063i	0.00004 -0.00063i

Contribution to w_+ at low-beta from all skew-quads circuits with integrated strength / quad K1L=10^{-5} (0.033 T)

Summary:

♦ The proposed SQC0 circuit complements the SQD0 circuit permitting a complete sum resonance correction both locally and globally.

♦ For the beam ellipse tilt correction at low beta only the SQD0 circuit is needed, SQC0 acts on the orthogonal component of w_+.

♦ However, SQD0 has equal effect at both main IPs. It is wothwhile to find a skew-quad (there is still a lot of unused ones) for differential correction of Rew_+ at B0 and D0.

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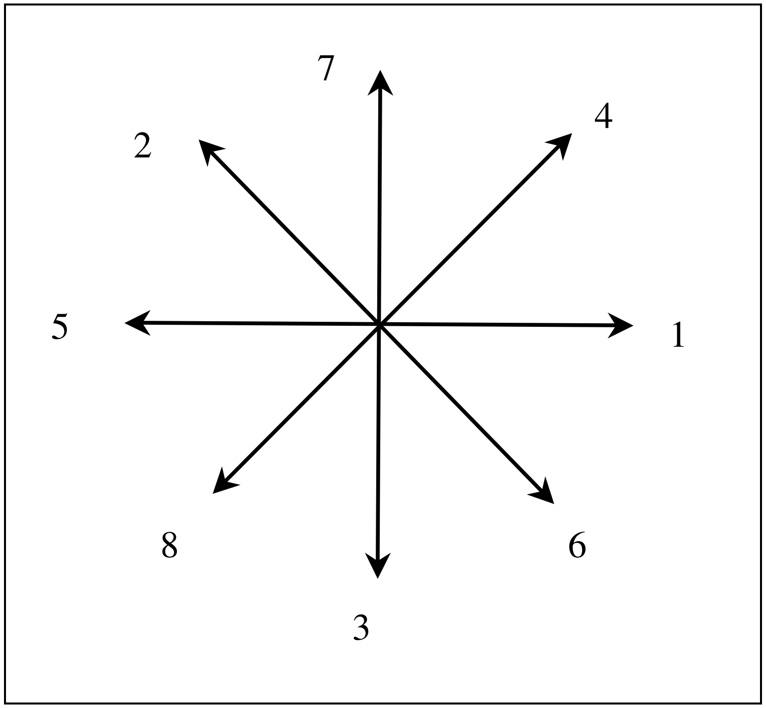
a_1 correction in dipoles:

There is 16 FODO cells/sextant in Tevatron, normally 8 dipoles/cell.
Phase advances per cell: μ_x=0.1870×2π, μ_y=0.1854×2π, $\mu_x + \mu_y$= 0.3724×2π ≈ 3π/4



In the case when all the dipoles have the same skew-quad gradient a_1, the contribution to w_+ from a group of 8 cells is intrinsically cancelled (see the Figure).

♦ Since each sextant comprises two groups by 8 cells it should not contribute to w_+

♦ Reshimming algorithm should be such that the selected dipoles occupied the same positions in groups by adjacent 8 cells.